S&V Questions and Answers

The Relationship Between % of Critical and Actual Damping in a Structure

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Question: **How is the "% of Critical" for a mode of vibration related to the actual damping in a structure?**

To understand the relationship of "% of Critical" to the actual damping in a structure we need to look at how damping manifests itself in Frequency Response Function (FRF) measurements. Modal analysis is used to characterized the dynamic properties of a structure. The dynamic properties of a structure are defined by its mode shape, damped natural frequency and damping for each mode of vibration over some frequency range of interest. These properties, the modal parameters, are determined from a set of FRFs that is collected at various points and directions of interest on the structure being analyzed. The FRF measurements are processed using curve-fitting algorithms that take the experimentally measured FRFs and fit them to an analytical function using a least squared error technique. The analytical function used during curve-fitting can be derived based upon the assumptions about the type of structure being analyzed. The assumptions of a linear 2nd order time invariant system allow us to derive the following equation for an FRF measurement:

$$H_{rs}(\omega) = \sum_{k=1}^{n}\left|\frac{R_{rs}^{(k)}}{(j\omega - p_k)} - \frac{R_{rs}^{*(k)}}{(j\omega - p_k^*)}\right|$$

where:

$H_{rs}(\omega)$ = FRF between response DOF r and excitation DOF s, g/lb

DOF = Degree of Freedom (DOF), the motion at a particular point in a particular direction, i.e., 3Z

ω = Frequency, rad/sec

n = Number of modes

$R_{rs}^{(k)}$ = Residue for response DOF r and excitation DOF s for mode k, g/lb-sec

p_k = Pole location for mode k=$-\sigma_k+j\omega$, rad/sec

σ_k = Damping decay rate or modal damping for mode k, rad/sec

ω_k = Damped natural frequency or modal frequency for mode k, rad/sec

This function defines the relationship of the modal parameters to the measured FRF measurement. The curve fitting process estimates the modal parameters (residue, modal frequency and modal damping) from the above equation or an equivalent form. How these parameters influence the response of a structure can more easily be shown from the impulse response function. The impulse response function can be determined from the inverse Fourier transform of the FRF. The Impulse response function is equivalent to the response of a structure to a Dirac-delta pulse forcing function. This is a theoretical input that has a width of zero and an amplitude of infinity. The impulse response function in terms of modal parameters is illustrated below:

$$h_{rs}(t) = \sum_{k=1}^{n}\left[\left|R_{rs}^{(k)}\right| e^{-\sigma_k t}\sin(\omega_k t + \phi_{rs}^{(k)})\right]$$

where:

$h_{rs}(t)$ = Impulse Response Function between response DOF r and excitation DOF s, g/lb

An impulse response measurement can be made by exciting a structure with an impact excitation. The resulting response will take on the general form of the impulse response function defined above. Inspecting the impulse response function above, one sees that the decay rate that controls the amplitude of the impulse response function is the modal damping σ_k.

To illustrate the effect of damping on a structure an FRF measurement was made on a lightly damped structure. This measurement was curve fitted to determine its modal parameters. The frequency and damping parameters are shown in Table 1.

The nondimensional critical damping fraction ζ_k is related to the modal frequency and modal damping by the following equation:

$$\zeta_k = \frac{\sigma_k}{\sqrt{\sigma_k^2 + \omega_k^2}} \qquad (0 \le \zeta_k \le 1)$$

It is common practice to express damping as % of critical, numerically equal to $100 \times \zeta_k$. As you can see from the above equation the "% of Critical" is a function of both the damping and damped natural frequency for a given mode of vibration. When evaluating damping changes made to a structure one must be careful when comparing "% of Critical" values because they are functions not only of the damping but also the damped natural frequency. One could envision a situation where a modification of a structure causes the damping and damped natural frequency to increase but the "% of Critical" value could decrease. When evaluating damping changes the modal damping σ_k is a direct measure of that modes' decay rate.

The lightly damped structure used to measure the FRF in Figure 1 was treated with a damping material and another FRF measurement was made. Table 2 shows the resulting changes in the pole locations (damped natural frequency and damping) of the modified structure.

The pole locations for the two sets of measurements are shown by the S-plane plot in Figure 2. Notice that the applied damping treatment added almost pure damping to the structure since the damped natural frequencies of the modes did not change significantly. Another comparison of the effect of the damping treatment can be seen in the impulse response function measurements for the unmodified and modified structure (see Figures 3 and 4).

Next month's Q & A column answers the question: **What is dynamic range and how do I improve it in my sound and vibration measurements?**

Send your questions or comments to:

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Table 1. Frequency and damping of lightly damped structure.

Mode	Frequency (Hz)	Damping (Hz)	Damping (%)
1	438	0.53	0.12
2	636	0.72	0.11
3	1335	1.80	0.13
4	1407	0.89	0.06
5	1847	2.11	0.11

Table 2. Frequency and damping of damped structure.

Mode	Frequency (Hz)	Damping (Hz)	Damping (%)
1	434	1.53	0.35
2	636	2.14	0.34
3	1336	4.34	0.32
4	1406	2.07	0.15
5	1845	3.79	0.21

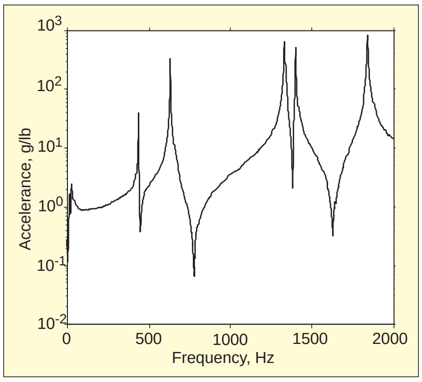

Figure 1. Frequency response function of lightly damped system.

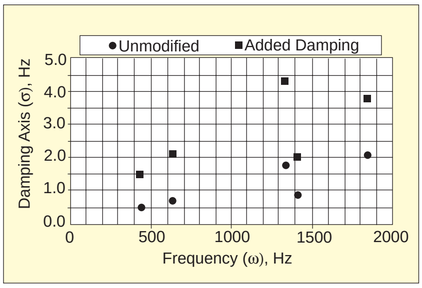

Figure 2. S-Plane plot comparing pole locations of an unmodified and modified structure.

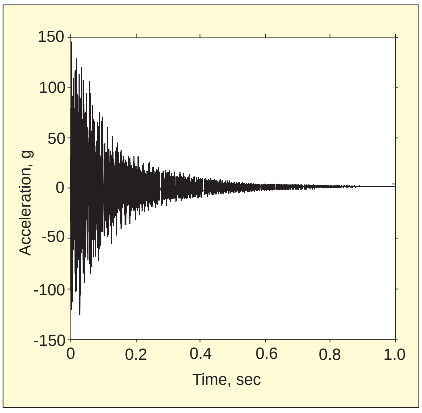

Figure 3. Impulse response of lightly damped structure.

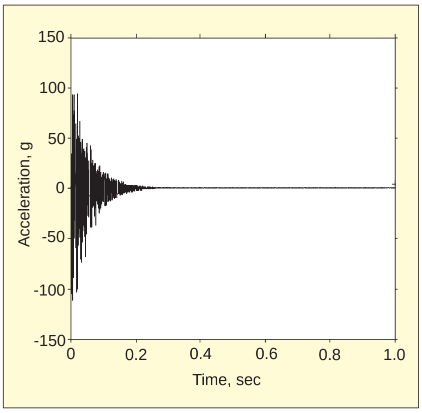

Figure 4. Impulse response of damped structure.