S&V Questions and Answers

What is the Coherence Function and How Can It Be Used to Find Measurement and Test Setup Problems?

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This month's column was going to be a continuation of the issues raised in last month's column regarding the repeatability of making Frequency Response Function (FRF) measurements using hammer/impact/tap testing techniques. However due to the length of the column we decided to break it up into two parts. In this month's column we will discuss the definition and use of the Coherence Function. Next month we will use the Coherence Function and a special impact test setup to address the repeatability and testing issues previously raised.

Question: **What is the Coherence Function γ^2 and how can it be used to find measurement and test setup problems?**

Answer: The Coherence Function is a computed measurement that gives a measure of the linear dependence between two signals as a function of frequency. This function is of particular importance and usefulness when making FRF measurements. When making FRF measurements for Modal Analysis the assumption is that the system under test is linear and the resulting FRF measurements are therefore representative of the linear system. Since the FRF measurement represents the relationship between two signals, an excitation point and direction and a response point and direction, the Coherence Function can be used to determine the linearity of an FRF measurement. The Coherence Function calculation is defined by the following equation:

$$\gamma^2(f) = \frac{\left|G_{xy}(f)\right|^2}{G_{xx}(f)G_{yy}(f)}$$

Where:

$G_{xy}(f)$ = Cross Power Spectrum between the excitation and response signal

$G_{xx}(f)$ = Power Spectrum of the excitation signal

$G_{yy}(f)$ = Power Spectrum of the response signal

f = frequency

The first thing to notice about the Coherence Function is that it is a function of frequency. Therefore its value can change depending on the frequency where the Coherence Function is evaluated. Notice also that the units of the Coherence Function are dimensionless as can be shown by substituting in common dimensions for a typical FRF measurement. For example assume the excitation signal used to excite the structure is calibrated in terms of pounds force (lbs) and the response is calibrated in terms of acceleration (g). Substituting these units into the equation for the Coherence Function yields:

$$\gamma^2 [=] \frac{[(\text{lbs})(\text{g})]^2}{(\text{lbs})^2(\text{g})^2} [=] \frac{(\text{lbs})^2(\text{g})^2}{(\text{lbs})^2(\text{g})^2} [=] 1$$

Another thing to notice about the Coherence Function is that it is a 'real' valued function since the numerator is the magnitude squared of the Cross Power Spectrum, a 'real' valued spectrum, and the denominator is the product of the Input (Force) Power Spectrum and the Response (Acceleration) Power Spectrum, both of which are 'real' valued spectrums. Therefore the division of these 'real' valued spectrum results in the Coherence Function being a 'real' valued spectra.

For those readers familiar with statistics you will notice that the form of the Coherence Function is very similar to the Pearson Correlation Coefficient r definition:

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}}$$

Where:

x = input variable

y = output variable

The Correlation Coefficient r is a measure of the degree of correspondence or relationship between two variables (for example Force and Acceleration). Correlated variables are those, which tend to vary together. When one is larger, the other tends to be systematically larger or smaller. Two variables are positively correlated when one variable being large causes the other variable to also be proportionately large too. They are negatively correlated when one is large and the other becomes corresponding smaller. The range of values for the Correlation Coefficient r is $-1 \le r \le 1$. When the two variables are perfectly positively correlated, a direct linear relationship exists such that a higher value on one variable (force) is always associated with a proportionately higher value (acceleration) on the other variable. This perfect linear positive correlation will have a coefficient of one (1). When two variables are perfectly negatively correlated, an inverse relationship exists and the correlation coefficient will be a minus one (–1).

The Coherence Function γ^2, when evaluated at a specific frequency, can be interpreted in a similar fashion as the square of the Correlation Coefficient r. The value of the Coherence Function γ^2 at any frequency can have a range of values between 0 (zero) and 1 (one). When the Coherence Function value is 1 there is a perfect linear relationship between the two signals of force and acceleration. When the Coherence Function is 0 there is no relationship between the two signals. In reality when making FRF measurements on physical systems the corresponding values of a Coherence Function γ^2 is somewhere between 0 and 1. Many of the systems we test are fairly linear so the resulting values of the Coherence Function will be very close to or equal to the value of 1. Figure 1 illustrates various Coherence Function values for different relationships that might exist between an excitation force and the corresponding response acceleration at a particular frequency, such as 1500 Hz. Each dot in Figure 1 represents the input/output relationship for one of the constituents used to compute the average FRF measurement at the frequency of 1500 Hz. The slope of the curve is actually the magnitude of the FRF measurement $|h|$, for example at a frequency of 1500 Hz.

In Figure 1a there is a perfect linear relationship between the excitation force and response acceleration therefore γ^2 at 1500 Hz = 1. In Figure 1b there appears to be a somewhat linear relationship with random spread due to added noise, which causes the γ^2 at 1500 Hz to be < 1. Figure 1c similarly shows the γ^2 at 1500 Hz to be < 1 but now it's due to a nonlinear relationship between the systems excitation and response. Finally in Figure 1d there is no relationship whatsoever between the excitation and response at 1500 Hz and therefore γ^2 = 0.

Did you ever wonder why the Coherence Function γ^2 was equal to unity at all frequencies for an FRF measurement consisting of only one average? It's because with only one average (estimate) of the excitation and response the system appears perfectly linear and therefore γ^2 = 1 at all frequencies.

Using Figure 1c we can also see another problem when using certain excitation techniques. If for example the excitation technique always excites the structure at the same force level, at a certain frequency for each average, the structure will respond with the same response. And if the system under test does not have very much noise on the input or output the Coherence Function γ^2 can be very close to a value of 1 indicating a linear system. However the system could be very nonlinear as is shown in Figure 1c but it appears to be linear because only one force level was used. Thus Figure 1c would only show one point at the particular force and response levels implying a perfect linear relationship. To determine whether a system is nonlinear or not, you should use an excitation technique whose force level will vary from one average to the next. This is why Pseudo-Random and Swept Sine excitation techniques are not very good for nonlinear systems.

The following lists several reasons why the Coherence Function may be less than unity at any frequency:

1. The presence of uncorrelated noise on one or both of the excitation and re-

sponse signals.

2. A true nonlinear relation between the excitation and response, i.e. a nonlinear structure. Note that a structure might be linear in certain frequency regions and nonlinear in others.
3. Leakage due to errors in the Fast Fourier Transform (FFT) algorithm when analyzing nonperiodic or truncated time waveforms.
4. Time delays between the excitation and response signals. For example using a low pass filter on the response signal to filter out an unwanted high frequency response could cause this. The filter causes an additional time delay and if large enough it will result in $\gamma^2 < 1$.

I frequently hear testers say that the Coherence Function is a measure of the causality between a system's excitation and response. I believe a more correct way to describe the Coherence Function is to say that it is a measure of how linearly related the response of the structure is to the excitation force.

Another point is that just because $\gamma^2 < 1$ at one or more frequencies does not necessarily mean that it's a bad measurement. It can be shown that if the bad coherence is due to noise that you just need to take more averages to have the same statistical confidence about the measured FRF than if the coherence is equal to 1. If the low coherence is due to nonlinear structural behavior then you must decide how to best characterize the structure's response to varied force levels. It should be noted that when $\gamma^2 < 1$ due to a nonlinearity, no amount of additional averaging will cause γ^2 to equal 1.

Since this is the last column of 1999 and marks the end of 1 year's worth of Q&A columns I thought it would be appropriate to thank and give credit to all the people that help put this column together. First I'd like to thank my colleagues at Sage Technologies who helped coauthor many columns and were always available for technical advice and guidance. Thanks go to Jack Mowry and his crew for their illustrating and formatting of the columns using the really raw material I would send as a final draft. Thanks to all for their constant and sometimes never ending stream of suggestions, critiquing and corrections.

Next Month's column will continue on with the issues raised in the November 1999 column regarding the repeatability of making FRF measurements using the hammer/impact/tap testing technique. While writing that month's column I was surprised at the apparent variance of the FRF measurements when being careful to make repeatable impact measurements. The variance could have been caused by a number of factors such as discussed in the October column or by nonlinear behavior of the test specimen. In next month's column we will use a test setup specifically designed to make repeatable impacts with controlled force amplitudes. This will allow us to better understand the variations in FRF measurements and the impact of averaging when we use impact-testing techniques. We will also use this opportunity to illustrate the use of the Coherence Function that was discussed above.

Next Month's Question: What are the sources of variations in Frequency Response Function (FRF) measurements using impact-testing methods and what impact does averaging have on this variation?

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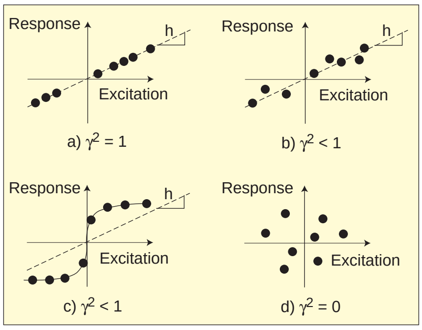

Figure 1. Coherence function values.