S&V Questions and Answers

The Exponential Window

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Question: **When should I use an exponential window? How does the exponential window work and can its effects on my data be removed by post-processing?**

You should only use an exponential window when your test object and data acquisition parameters require extra damping in the system to avoid the effects of leakage (more on this later). The exponential window is one of the few windows whose effects can be removed by post-processing and we will describe how to determine the amount of damping added by the exponential window and how to remove this added damping from the estimated damping in your Modal Analysis System

Determining Whether You Need an Exponential Window. Exponential windows, when needed, are only applied to motional responses in reaction to impulsive inputs like hammer tests. If you are using a nonimpulsive signal source such as a shaker with random or sine excitation, you should not use an exponential window.

During impact testing, leakage will occur whenever the analyzer's time record capture length is shorter than the time required for the response channel to decay to an amplitude (near zero) below the analyzer's noise floor. Figure 1 shows a measured response signal from an accelerometer where the analyzer did not completely measure the response. The response signal has been truncated which will result in errors when the response time history is transformed into the Frequency Domain using the Fast Fourier Transform (FFT) algorithm. These are called leakage errors and will show up in both amplitude and frequency parameters of the test results. (The effects of leakage using various excitation techniques will be a topic of a future Q&A column.) The effect of this leakage error on a measured FRF is illustrated in Figure 2.

Eliminating Leakage without Using the Exponential Window. Leakage can be eliminated in one of two ways when using impact testing to measure FRFs. The first and preferable way is to set up the analyzer such that its Time Record length is sufficiently long enough to capture the complete response time history of the response signal. This is accomplished by making the frequency resolution Δf of the measurement smaller. The frequency resolution Δf is improved by increasing the number of frequency lines of resolution or by using a smaller frequency analysis bandwidth. Optimally, the response channel time history should decay to zero at the end of the analyzer's Time Record. This would result in the following optimum frequency resolution required to make a good FRF measurement:

$$\Delta f = \frac{1}{T}$$

where:

Δf = analyzer's frequency resolution, Hz

T = analyzer's time record length, sec.

This is preferred because no special signal processing windows need be applied to the response channel. This results in a very accurate measurement that has no leakage errors or distortion caused by the signal processing window. The same FRF measurement that was illustrated above was measured again in Figures 3 and 4 after the analyzer was set up to capture the response channel's entire time history. Notice that the amplitude and shape of the leakage free measurement is dramatically different from that in Figure 2.

Using an Exponential Window to Eliminate Leakage. The second way to eliminate or minimize leakage is to apply an exponential window. We use signal processing windows much of the time to help minimize leakage errors. Signal processing windows are applied in the Time Domain by multiplying the measured signal $y(t)$ by some special time varying function $w(t)$. Some of the more common window functions have familiar names, such as: Hanning, Rectangular, Flat Top and Exponential. The windowing operation of the analyzer can be expressed by the following equation:

$$y'(t) = w(t)\, y(t)$$

where:

$y'(t)$ = modified time history (windowed)

$w(t)$ = signal processing window

$y(t)$ = analyzer's original sampled time history.

The exponential window is ideally suited for impact testing because of its predictable effects on the resulting FRF measurement; more on that later. The equation of the exponential window is:

$$w(t) = e^{-(t/\tau)} = e^{-\sigma_0 t}$$

where:

τ = exponential time constant, sec

σ_0 = damping decay rate, rad/sec ($1/\tau$).

When the exponential window is applied, it attenuates the amplitude of the response signal exponentially from a factor of 1.0 to a small value. The ending value is a function of the window's time constant τ and the analyzer's time record length T. The exponential window has a predictable effect on the response time history because of the nature of the structure's response from an impact forcing function. A structure responds as a summation of decaying sinusoids when an impact is used as the excitation force. The general form of the response of a structure being excited by an impact is:

$$y(t) = \sum_{k=1}^{n} \left[R_k e^{-\sigma_k t} \sin(\omega_k t) \right]$$

where:

n = number of modes in the frequency range being excited

R_k = constant for mode k

σ_k = damping decay rate for mode k

ω_k = damped natural frequency for mode k.

When an exponential window is applied, the response time history becomes:

$$y'(t) = w(t)\, y(t)$$

$$y'(t) = e^{-\sigma_0 t} \sum_{k=1}^{n} \left[R_k e^{-\sigma_k t} \sin(\omega_k t) \right]$$

$$y'(t) = \sum_{k=1}^{n} \left[R_k e^{-(\sigma_0 + \sigma_k)t} \sin(\omega_k t) \right]$$

$$y'(t) = \sum_{k=1}^{n} \left[R_k e^{-(\sigma'_k)t} \sin(\omega_k t) \right]$$

where the apparent damping of mode k is:

$$\sigma'_k = \sigma_0 + \sigma_k$$

From the above equations, one can see that the exponential window does nothing more than add a constant amount of damping σ_0 to each mode of vibration in the FRF measurement. The modal analysis system processes the set of FRF measurements and the apparent damping σ'_k is estimated during the curve fitting process. If you knew how much damping was added to each mode from the use of the exponential window, you could calculate the modes actual damping using the following equation:

$$\sigma_k = \sigma'_k - \sigma_0$$

Setting the Exponential Window on Your Spectrum Analyzer. So the issue now becomes how to determine the damping decay rate σ_0 of the exponential window that has been selected. Unfortunately analyzer manufacturers do not implement the setting of the exponential window parameters in a common manner. However, most use one of two methods to directly or indirectly specify the amount of damping σ_0 added to an FRF measurement.

One method is to specify the exponential time constant τ by entering a value or positioning a cursor that specifies the value of the time constant. The time constant τ is the time required for the exponential to decay to a value of 0.368 as shown by the equation below:

$$w(\tau) = e^{-(t/\tau)}\Big|_{t=\tau} = e^{-(\tau/\tau)} = e^{-1} = 0.368$$

In other words, the response signal has been attenuated by 63.2% when $t = \tau$. After τ is known, you can determine the amount of damping from the equation:

$$\sigma_0 = \frac{1}{\tau}$$

For example, if τ = 0.1 sec then the amount of damping added would be σ_0 = 10.0 rad/sec = 1.592 Hz.

The second method is to specify the value of the exponential window at the end of the analyzer's time record T, such as 0.10. This would imply that at the end of the analyzer's time record, the exponential window has attenuated the response signal by 90%. In order to compute the damping σ_0 added, you need to determine the value of the analyzer's current time record length T. Some analyzers will report this value directly or you can compute it using the current Δf of the measurement $T = 1/\Delta f$. Once T is known, the damping σ_0 can be calculated from the following equation:

$$\sigma_0 = -\frac{\ln[w(T)]}{T}$$

where:

σ_0 = damping decay rate, rad/sec
$w(T)$ = value of exponential window at end of analyzer's time record
T = analyzer's time record length, sec.

For example, if the value of the exponential window is 0.10 at the end of a time record of 0.64 sec, the damping σ_0 will be:

$$\sigma_0 = -\frac{\ln(0.10)}{0.64} = 3.598 \text{ rad / sec } = \text{ } 0.573 \text{ Hz}$$

Consequences of Using an Exponential Window. There is a down side to using the exponential window. As more artificial damping is added, closely spaced modes get smeared together and the more difficult the curve fitting process becomes. Figure 4 shows an FRF measurement that was made without using an exponential window and no leakage. Contrast this with the measurements made in Figures 5 and 6 where two different exponential windows were used. Figure 5 shows the effects of an exponential window that decayed 90% and Figure 6 similarly shows the effects of one that decays 99%. Notice that the peaks in the FRFs are somewhat broader and lower in amplitude.

Modal analysis systems will display a frequency and damping table after the curve fitting process. If an exponential window was used when collecting the set of FRF measurements, the damping frequency σ'_k estimated for each mode becomes too large by the amount σ_0. One can then simply subtract the added damping σ_0 from the estimated damping σ'_k to determine the actual damping σ_k.

This sounds good, but in practice does not work very well. Many times this calculation results in a negative damping value because of errors in the measurement and/or curve fitting process. Typically the values of the actual damping σ_k are relatively small as compared with the amount of damping added σ_0. Thus, when subtracting two relatively large values to determine a small value, the result can easily be negative when one of the larger values is in error.

Next month's Q & A column answers the question: **When should I use a free field, random incidence or pressure microphone. What errors will I incur if I use the wrong one?**

Send your questions or comments to:

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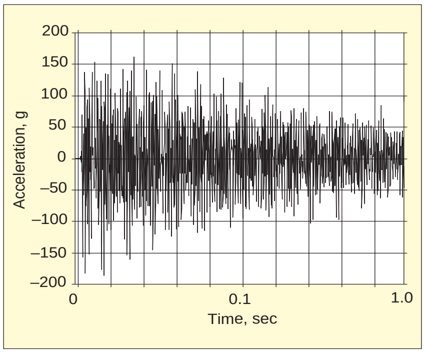

Figure 1. Truncated response time history.

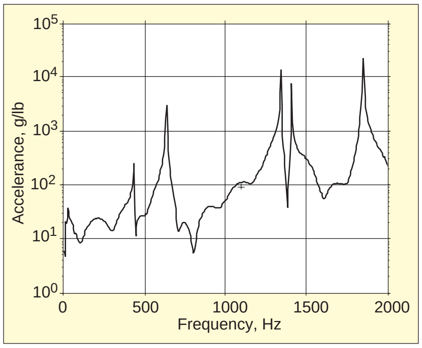

Figure 2. Frequency response function with leakage.

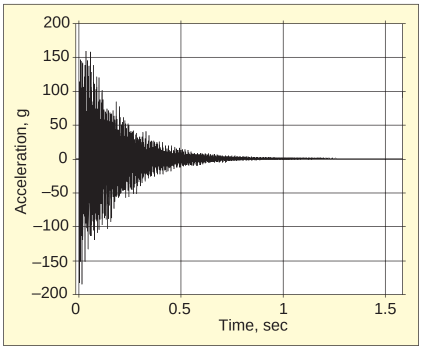

Figure 3. Completely captured response time history.

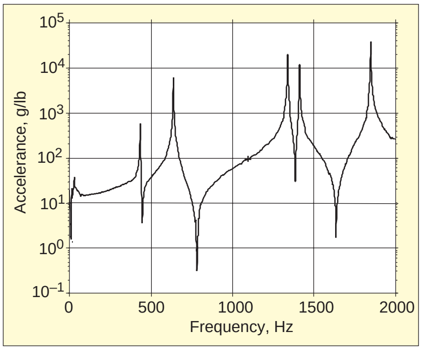

Figure 4. Frequency response function without leakage.

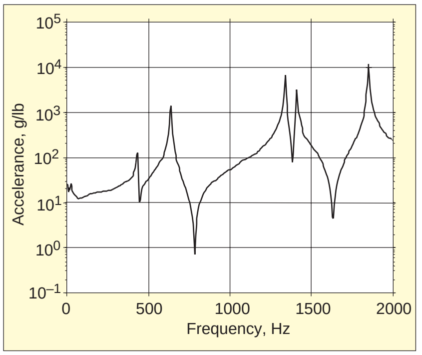

Figure 5. FRF using exponential window, 90% decay.

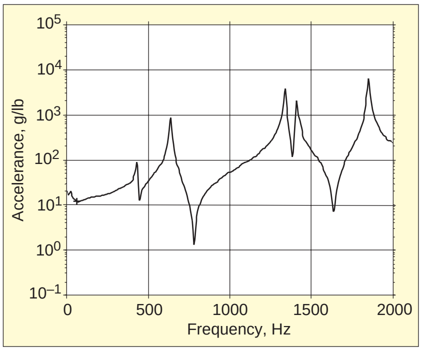

Figure 6. FRF using exponential window, 99% decay.