S&V Questions and Answers

What Are the Sources of Variations in Frequency Response Function Measurements?

David Formenti, Sage Technologies, Morgan Hill, California

This month's column will continue on with the issues raised in last November's column regarding the repeatability of FRF measurements made using the hammer/impact/tap testing technique. While writing November's column I was surprised at the apparent variance of the FRF measurements when being careful to make repeatable impact measurements. The variance could have been caused by a number of factors such as discussed in November's column or by nonlinear behavior of the test specimen. In this month's column we will use a test setup specifically designed to make repeatable impacts with controlled force amplitudes. Hopefully this will allow us to better understand the variations in FRF measurements and the effect of averaging when we use impact-testing techniques. We will also use this opportunity to discuss the use of the Coherence Function.

Question: **What are the sources of variations in Frequency Response Function (FRF) measurements using impact-testing methods and what effect does averaging have on this variation? How is the Coherence Function interpreted and used to find measurement/setup problems?**

Answer: In November we demonstrated that one big source of variation when making FRF measurements is the tester's ability to impact the same point in the same direction. While averaging multiple FRF measurements together it is assumed that each measurement in the average is for the same excitation and response degrees-of-freedom. When this is not the case, the resulting averaged FRF will be in error. We illustrated this by varying the point and direction of impacting while averaging and compared these measurements to an FRF measurement made with 'repeatable' impacts. Even though the FRFs made with repeatable impacts had much less variation, they still covered a surprising range of values.

In order to better understand variations in FRF measurements, a special impact-testing test setup was designed. The test setup, shown in Figure 1, is designed to make repeatable impacts while averaging an FRF measurement. The test setup consists of a test item (aluminum I-Beam) suspended and held in place by nylon line and an instrumented impact hammer suspended with nylon line as a bifilar pendulum. This test setup allows for good control and repeatability of impact point and direction. Measuring the distance the pendulum is displaced prior to the impact can also accurately control the excitation force level.

An FFT spectrum analyzer was used to simultaneously measure two FRF measurements, the excitation point and direction was at 23Z and the two response measurements were at point 33 in the X and Z directions. The analyzer was set up to make a 'leakage' free zoom measurement with a frequency range of 900 to 1900 Hz. In this frequency range there were three modes of vibration at the approximate frequencies of 1338 Hz, 1411 Hz, and 1850 Hz.

The first test tried to quantify the variability of unaveraged FRF magnitudes. Figure 2 shows a typical pair of FRF measurements that resulted from one impact. This test was repeated for two different excitation force levels. Displacing the bifilar pendulum (hammer) an amount of 2.5 in. and 5.0 in. controlled the excitation force levels for the two tests. Each test at the different force levels consisted of a series of 10 FRF measurements with one average. Table 1 summarizes the results of these tests.

The results indicate that the test setup allowed for fairly repeatable measurements with the possible exception being the results for Mode 3 of the low force excitation FRF 23Z/33Z. The peak amplitudes of this mode had the largest variation by far for some unknown reason. However the rest of the results were very repeatable in terms of the two levels of excitation and the resulting peak amplitudes. The interesting result is that the peak amplitudes of all the modes for both of the FRF measurements increased with increasing excitation force level by roughly 10%. This increase in amplitude could be explained by small nonlinear behavior of the structure under test or nonlinear behavior of the transducers. If the system were totally linear, you would not expect to see any differences in the peak amplitudes of the FRFs at the different force levels. The response of the structure per unit force in Test 2 is more than that in Test 1, which might be indicative of a softening spring mechanism as the nonlinearity.

Figure 1. Experimental setup.

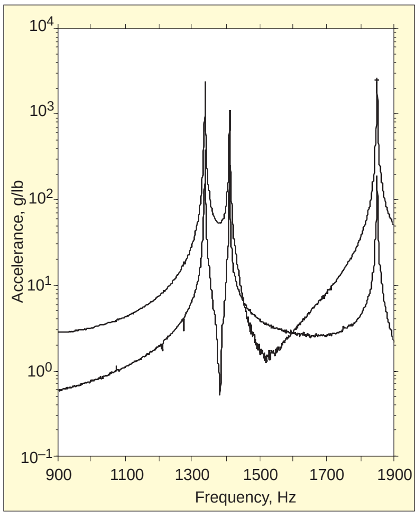

Figure 2. Typical FRF measurements 23Z/33Z and 23Z/33X.

A nonlinearity of the accelerometers was ruled out since two different accelerometers were used to measure the two FRFs (23Z/33X and 23Z/33Z) and both FRFs showed the same 10% variation in amplitude. This left only the structure or the hammer's load cell as the source of the nonlinearity. In order to eliminate the load cell, I repeated the set of experiments using a different transducer. The results of this set of tests revealed the peak amplitudes of the FRF measurement did NOT change when the force level was increased. Thus the structure was indeed linear with respect to load levels and that the nonlinear behavior measured in the first set of experiments was due to load nonlinearities of the load cell in the frequency range of the measurement.

The relatively large variation in values in last month's column I believe can now be explained by the uncontrolled amplitude of the excitation force that resulted from using the standard impact testing

Table 1. Summary of test results.

	Excitation Force lbs	23Z/33Z Mode 1 1338 Hz g/lb	Mode 2 1411 Hz g/lb	Mode 3 1850 Hz g/lb	23Z/33X Mode 1 1338 Hz g/lb	Mode 2 1411 Hz g/lb	Mode 3 1850 Hz g/lb
Test 1							
Average	1.8	2198.8	1081.4	1948	356.6	245.8	150.4
Standard Deviation	0.1	27.3	22.0	182.5	4.3	5.0	13.9
	Excitation Force lbs	**Mode 1 1338 Hz g/lb**	**Mode 2 1411 Hz g/lb**	**Mode 3 1850 Hz g/lb**	**Mode 1 1338 Hz g/lb**	**Mode 2 1411 Hz g/lb**	**Mode 3 1850 Hz g/lb**
Test 2							
Average	4.7	2407.6	1136.9	2451.0	388.5	258.6	188.3
Standard Deviation	0.3	24.8	16.3	18.6	3.9	3.7	1.2

technique and the nonlinear load behavior of the impact hammer's load cell. When using impact testing to measure FRFs it is very difficult to input a consistent excitation force from one average to the next. This is of particular importance when you are testing a structure that is load nonlinear. This just goes further to emphasize the importance of picking the right excitation technique to measure the dynamics of a structure and the importance of using transducers that are linear in the test range. Don't just assume impact testing is the only way to measure FRFs or that your transducers are linear within the test range.

Now let's look at how the Coherence Function can be interpreted to determine measurement problems associated with linearity and noise. In last month's column (December 1999) we defined the Coherence Function and how it was calculated. In that column we showed that the Coherence Function gives a measure of the linear dependence between two signals as a function of frequency. Now we'll look at why this function is of particular importance and usefulness when making FRF measurements. The Coherence Function calculation is defined by the following equation:

$$\gamma^2(f)=\frac{\left|G_{xy}(f)\right|^2}{G_{xx}(f)G_{yy}(f)}$$

Where:

$G_{xy}(f)$ = Cross Spectrum between the excitation and response signal

$G_{xx}(f)$ = Power Spectrum of the excitation signal

$G_{yy}(f)$ = Power Spectrum of the response signal

f = frequency

Remember that the Coherence Function is a function of frequency. Therefore its value can change depending on the frequency where it is evaluated.

Figure 3 shows a typical Coherence Function and FRF measurement that was made using the test setup in Figure 1. The top graph is the Coherence Function and the bottom graph is the magnitude of the FRF measurement. The measurements were made using a total of 5 averages.

Note that the Coherence Function graph has been expanded about the value of 1.0 to better show its detail. Notice that the Coherence Function is not perfect unity although it is very near the value of 1.0.The biggest dip in the Coherence Function occurs at the very deep valley (zero) of the FRF just below 1400 Hz. This dip is because of the relative large amount of noise present with respect to the low level of the response of the system at this frequency. In other words we can say that the signal to noise ratio in this frequency range is low. This causes the measurement to have a lower Coherence Function value because of noise. This is a common occurrence in measurements were the response of the structure is very small at particular frequencies.

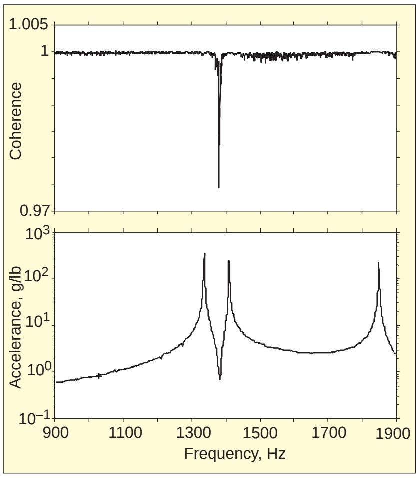

Figure 3. Typical Coherence Function and corresponding FRF, 23Z/33X.

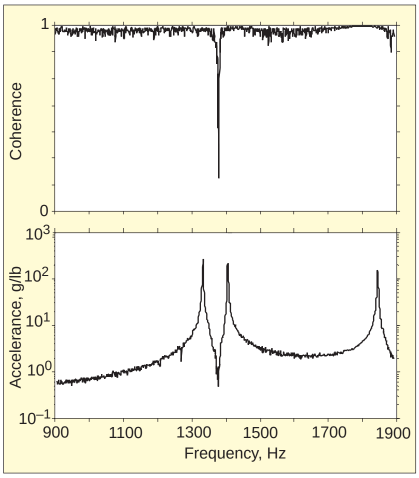

Figure 4. Effect of rattles on Coherence Function and corresponding FRF, 23Z/33X.

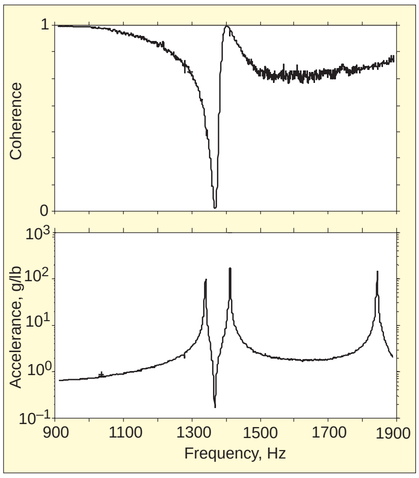

Figure 5. Effect of inconsistent excitation points and/or direction on Coherence Function and corresponding FRF, 23Z/33X.

Rattles cause another common measurement problem in the system. When a system has loose joints, parts or components, rattles can be excited from the input excitation. These rattles cause additional force inputs to the system that are not being measured. However the system responds to these inputs, which then gets measured by the response accelerometer. These rattles add noise to the system that will dramatically affect the quality of the Coherence Function and FRF measurement. Figure 4 illustrates this effect. This measurement was made using the same setup in Figure 1 except a coin was laid on the web of the I-Beam.

Note that the Coherence Function in Figure 4 has now been plotted over the entire range of values, 0 to 1. Typically you will hear the rattles during the setup phase of the test when you make trial FRF measurements. You should try to find them and stop the rattles by taping down loose parts or wedging paper into loose joints.

The Coherence Function will also help you evaluate how careful you are to always impact the same point in the same direction as you are averaging while measuring an FRF. Figure 5 shows how dramatically the Coherence Function senses what appears to be a nonlinear response of the structure due to an impact that is either at a different point and/or direction than the others in an average. The FRF measurement in Figure 5 was made using three averages. The first two averages were done by impacting 23Z (see the setup in Figure 1) and for the third average the excitation point was moved about 1 in. away from 23 but impacted in the same direction.

The interesting thing to note about Figure 5 is that the FRF looks very good. It's only when you inspect the Coherence Function that you get a hint that there may be a problem.

One final comment about the Coherence Function, at the beginning of this discussion we said that the Coherence Function gives a measure of the linear dependence between two signals as a function of frequency. We can use this fact directly to test a structure for load nonlinearity. Set up a trial FRF measurement using let's say three averages. While making the first two averages take care to impact the structure with the same force level. Then during for the final average measurement, impact the structure with either a much larger or much smaller force. If the structure is linear the Coherence Function should be close to a value of 1. If it is non-linear over any part of the frequency range, the Coherence Function value will generally be much smaller than 1 depending on how nonlinear the structure really is.

Next Month's Question: What is the difference between a 'Studio' microphone and a 'Measurement' microphone? Under what circumstances should you use one over the other?

Send your questions or comments to:

Dave Formenti
Sage Technologies
16675 Buckskin Court
Morgan Hill, CA 95037
Phone: (408) 776-1106
Fax: (408) 776-1107
Email: dformenti@thesagesite.com