S&V Questions and Answers

Excitation Direction in Modal Testing – Is It Important?

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Question: **When measuring Frequency Response Functions (FRFs) for the purpose of Modal Analysis, does it matter if a fixed response transducer (impact testing) or a fixed excitation (shaker testing) is aligned with any particular axis or arbitrarily positioned?**

Answer: No! The only exception would be if, for a particular mode, the mode shape component for that mode had an amplitude of zero (0.0) at the point and direction where the reference transducer (impact testing) or fixed excitation (shaker testing) was attached.

To further understand this we need to refer to the following definition of a Frequency Response Function (FRF). An FRF measurement of a linear, second order, time invariant system can be represented by its corresponding modal parameters using the following equation:

$$h_{ab}(\omega) = \sum_{k=1}^{n}\left[\frac{R_{ab}^{(k)}}{j\omega - p_k} + \frac{R_{ab}^{(k)^*}}{j\omega - p_k^*}\right]$$

where:

$h_{ab}(j\omega)$ = Frequency Response Function between response point and direction (a) and excitation point and direction (b)

ω = frequency

n = number of modes in frequency range of measurement

$R_{ab}^{(k)}$ = Residue for mode k between response point and direction (a) and excitation point and direction (b)

p_k = Pole location (damping decay rate and damped natural frequency) for mode k

The **modal parameters** are the complex constants $R_{ab}^{(k)}$ and p_k. The Residue can also be represented as the product of the mode shape components:

$$R_{ab}^{(k)} = u_a^{(k)} \cdot u_b^{(k)}$$

where:

$u_a^{(k)}$ = Mode Shape component for mode k at response point and direction (a)

$u_b^{(k)}$ = Mode Shape component for mode k at excitation point and direction (b)

The point and direction (a and b) are generally referred to as degrees-of-freedom (DOF). Typically these DOFs are defined with respect to a particular coordinate axis. For example, the DOF of a = 5z corresponds to motion at point 5 in the z-direction.

Now let's look at an example using three FRF measurements that were made on a structure using the classical impact testing technique with a fixed response reference transducer. The reference transducer in this case is mounted to the structure at a point in some arbitrary direction that is not aligned with any global or local coordinate system with respect to the structure. The measured FRFs were:

$$h_{m,10z};h_{m,11z};h_{m,12z}$$

where:

m = the DOF where the reference response transducer was attached

10z,11z,12z = the DOFs of the roving hammer excitation

Using the above definition for a FRF measurement the corresponding Residues for the above measurements for the k^{th} mode are:

$$R_{m,10z}^{(k)};R_{m,11z}^{(k)};R_{m,12z}^{(k)}$$

These Residues can be further expressed in terms of the mode shape components at the different DOFs for the three (3) FRF measurements.

$$R_{m,10z}^{(k)} = u_m^{(k)} \cdot u_{10z}^{(k)}$$
$$R_{m,11z}^{(k)} = u_m^{(k)} \cdot u_{11z}^{(k)}$$
$$R_{m,12z}^{(k)} = u_m^{(k)} \cdot u_{12z}^{(k)}$$

Here we see that the Residues are made up of the various mode shape components $u_{10z}, u_{11z}, u_{12z}$, etc. multiplied by the component of the mode shape corresponding to the m-DOF.

Many of the modal analysis systems on the market today use the estimated Residues, determined from curve fitting the FRFs, directly to display the animated mode shapes of a structure. This is an acceptable way of displaying a mode shape, because the Residues are really the mode shape of interest, multiplied by some scalar constant. (Note that this is only true if the same reference response location is used to measure all of the FRF measurements.)

In the case of an impact test, the scalar constant is the mode shape component of a particular mode at the point and direction of the reference response transducer, whether or not the reference response transducer is aligned to some coordinate axis. Thus, in the above example this scalar constant would be u_m. So, you can see that if you were very unlucky and selected a point and direction to place your reference response transducer exactly at a node for a mode (a node is where the mode shape component is 0 for a particular direction), the mode would not appear in any of the FRF measurements. This is because u_m = 0 for that mode and hence the corresponding Residues would be zero and the mode would then be absent from all measured FRFs.

This same reasoning can be applied to shaker testing. When measuring FRFs using a shaker to excite a structure we many times attach the shaker to be aligned with a global coordinate system. However, this is not a requirement of the testing technique. One can use the same reasoning as in the above example to show that as long as the point and direction of the excitation force do not change during the modal test and are not at a mode's node, the shaker can be mounted to the structure in any arbitrary direction.

Next Month's Question: What is an L_{eq} measurement when using an Integrating Sound Level Meter to measure sound level?

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