

Is It Important to Hit the Same Point in the Same Direction When Making Impact FRF Measurements?

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Question: When making an impact (hammer) type of Frequency Response Function (FRF) measurement (sometimes referred to as tap testing) with several averages, is it important to hit the same point in the same direction for each average? If so, why?

Answer: YES! In general you must always apply the excitation in the same direction because you are averaging together a series of frequency response estimates. The FRF measurement is made between two Degrees of Freedom (DOFs). One DOF is the excitation point and direction and the other DOF is the response point and direction. An averaged FRF is the result of the system dynamics between the excitation DOF and the response DOF.

A transfer function of a linear, second order, time invariant system can be represented by its corresponding modal parameters using the following equation:

$$h_{ab}(s) = \sum_{k=1}^n \left[\frac{R_{ab}^{(k)}}{s - p_k} + \frac{R_{ab}^{(k)*}}{s - p_k^*} \right]$$

where:

$h_{ab}(s)$ = transfer function between response point and direction (a) and excitation point and direction (b)

s = complex Laplace variable

n = number of modes in frequency range of measurement

$R_{ab}^{(k)}$ = residue for mode k between response point and direction (a) and excitation point and direction (b)

p_k = pole location (damping decay rate and damped natural frequency) for mode k .

The modal parameters are the complex constants $R_{ab}^{(k)}$ and p_k . The residue can also be represented as:

$$R_{ab}^{(k)} = u_a^{(k)} \cdot u_b^{(k)}$$

where:

$u_a^{(k)}$ = the mode shape component for mode k at response point and direction (a)

$u_b^{(k)}$ = the mode shape component for mode k at excitation point and direction (b).

The point and direction (a & b) are generally referred to as degrees-of-freedom (DOFs). Typically these DOFs are defined with respect to a particular coordinate axis. For example, the DOF of $a = 5z$ corresponds to motion at point 5 in the z -direction.

The frequency response function that

we now measure experimentally can be shown to be the transfer function, as defined above, evaluated for $s = j\omega$, where $j = \sqrt{-1}$ and ω is the frequency in radians/sec. In the above equation, if we substitute $j\omega$ for s we will have the equation of a frequency response function (FRF).

During the actual measurement process, several estimates of an FRF are essentially averaged together (actually it's the cross power spectrum between DOFs a & b and the auto power spectrum of DOF b that are averaged). Since the residues, and thus the mode shape components, appear in the equation of the FRF in a linear fashion, the residue of the average FRF will be the average of all the residues from each FRF estimate for each mode. As we have shown above, the residue for a given mode of vibration is a function of the mode shape components at the two DOFs (a & b).

It is also important to note that the pole location of a mode (p_k) is *not dependent* on the location of the FRF measurement because the pole is a global parameter. The residue, however, is not only a function of the particular mode of interest, but also a function of the response (a) and excitation (b) DOF.

When performing a hammer (impact/tap) test, the reference transducer is typically attached to the structure at a fixed point in a particular direction. This will be referred to as DOF (a). During the measurement this reference transducer stays fixed, so it is always measuring the same portion of a residue for a particular mode, namely u_a . However, if, from one measurement to another during the averaging process, the excitation to the structure happens to be at a different direction and/or point, that portion of the residue as defined by the excitation DOF u_b , will be used in making up the residue for that measurement, instead of the desired u_b .

For example, suppose we wish to measure a frequency response measurement using two averages. The average FRF \hat{h}_{ab} would essentially be computed as:

$$\hat{h}_{ab} = \frac{(h_{ab} + h_{ab'})}{2}$$

where the subscripts have the following meaning:

a = DOF of the fixed response accelerometer, i.e., 33x

b = DOF of the excitation (hammer), i.e., 1z

b' = DOF of the excitation (hammer), i.e., 1z', where z' is off by some angle θ from being coincident with the z axis of the global co-

ordinate system.

Now, let's look at the residue for mode 1 of both of the measurements that make up that particular average.

$$\hat{R}_{ab}^{(1)} = \frac{(R_{ab}^{(1)} + R_{ab'}^{(1)})}{2}$$

Again remember that the pole location p_1 (for mode 1) is the same as measured from DOF b or b' .

The average residue above, can be further expressed in terms of the mode shape components as follows:

$$\hat{R}_{ab}^{(1)} = \frac{(u_a^{(1)} \cdot u_b^{(1)} + u_a^{(1)} \cdot u_{b'}^{(1)})}{2} = \frac{u_a^{(1)}(u_b^{(1)} + u_{b'}^{(1)})}{2}$$

Thus, from the above relationship we see that if the two mode shape components at DOF b and b' are different (which in general will always be the case), the average residue we calculate from the average FRF will be in error, because we would be averaging two different mode shape components together. If DOFs b and b' are very nearly the same, then the two corresponding mode shape components will be also very nearly the same and thus the error will be small.

This is generally what happens when we are testing in the low frequency range where the mode shape components do not drastically change in the vicinity of a particular impact point and direction. However, when we do testing in the higher frequency range the mode shape components can change quite dramatically in the vicinity of where we are making the impact. Hence it is always good practice to impact the same point in the same direction for each of the averages in a frequency response function measurement.

To illustrate this potential problem, I set up a structure (aluminum I-beam, 3" x 4" x 10") for impact testing and picked an FRF that involved two different coordinate directions, an excitation at 1z and a response at 33x. I made 5 FRF measurements with one average each. The family of 5 measurements in Figure 1 represents a set of FRFs that were carefully made by impacting point 1 in the z -direction correctly for each measurement.

The FRFs in Figure 1 have been plotted over a narrow frequency range to include only 2 modes to better view the differences in peak amplitude. The interesting point to be made in Figure 1 is that even though the measurements were carefully made there is still a variation in peak amplitude for both modes from one measurement to the next. This variation in amplitude is still probably due to the small variations in the excitation point and direction. Even with care the variation in amplitude is 14% for the first peak and 15% for the second. If these measurements were averaged together, which normally would be the case, the average amplitude of the first peak would be 4210 g/lb and the second peak 1258 g/lb.

Figure 2 is another family of measure-

ments made under the same conditions as above except that the excitation point was varied while trying to maintain an accurate z-direction. The various excitation points used were all within approximately 1/4" of point 1, used for the FRF measurements in Figure 1.

The variance in the FRF measurements in Figure 2 comes from the fact that the peak amplitudes are highly dependent on the product of the mode shape components between the excitation DOF and response DOF. When the excitation point varies from average to average, the mode shape component at the excitation DOF for all the modes measured also changes. The peak amplitude for the first mode varied by 30% and for the second mode by 38%. If these measurements were averaged, the first peak would have an average value of 3409 g/lb and the second peak would be 1058 g/lb.

Finally, Figure 3 illustrates what happens when the direction is varied while exciting the same point. Here again the peak amplitude variation is primarily due to the variation in the excitation DOF mode shape component as the direction of impact changes. These measurements were made by exciting point 1 using impacts that were at approximately a 30 degree angle with respect to the vertical z-axis. The variation in peak amplitude for the first peak is 23% and 36% for the second peak. If averaged, the first peak would be 3992 g/lb and 1226 g/lb for the second. Table 1 summarizes the differences between the 3 different impact tests illustrated in Figures 1-3.

The conclusion is that when doing impact testing you should take care to always excite the structure at the same point in the same direction for each average. This becomes even more important if you plan to use the set of FRF measurements to build a model for structural dynamics modification investigations.

Next month's column will continue on with the issues raised in this column regarding the repeatability of making FRF measurements. While preparing this month's column, I was surprised at the apparent variance of the FRF measurements when being careful to make repeatable measurements. This variance could be caused by a number of factors such as discussed here or by nonlinear behavior of my test specimen. Next month we will use a test setup specifically designed to make repeatable impacts with controlled force amplitudes. Hopefully this will allow us to better understand the variations in FRF measurements and the impact (no pun intended) of averaging when we use impact testing techniques. We will also use this opportunity to discuss the use of the Coherence Function.

Next month's question: What are the sources of variations in FRF measurements using impact testing methods and what impact does averaging have on this variation? Additionally we will discuss

how the Coherence Function is interpreted and how this function can be used to find measurement/setup problems.

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Table 1. Summary of results.

	Amplitude of Peak, g/lb	
	First Mode	Second Mode
No Variation		
Maximum	4521	1371
Minimum	3864	1157
Average	4210	1258
Standard Deviation . . .	235	76
Varied Point		
Maximum	4116	1345
Minimum	2885	836
Average	3409	1058
Standard Deviation . . .	551	230
Varied Angle		
Maximum	4448	1445
Minimum	3412	931
Average	3992	1226
Standard Deviation . . .	379	230

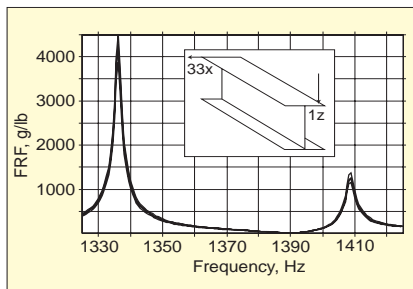


Figure 1. Family of 5 FRFs correctly impacted in the 1Z direction.

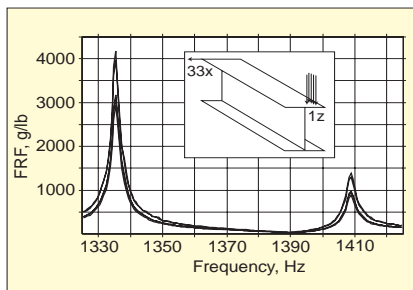


Figure 2. Family of 5 FRFs impacting in the vicinity of point 1.

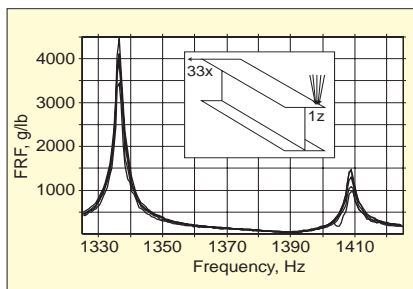


Figure 3. Family of 5 FRFs impacting at point 1 at angles to the Z-axis.