S&V Questions and Answers

Do Mode Shapes Determined from Two Different Impact Testing Techniques Contain the Same Information?

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This month's column is about analyzing the mode shapes2 of a structure using two different impact testing techniques to collect Frequency Response Functions (FRFs). One technique uses a fixed reference triaxial accelerometer while impacting various points of interest in only the *z*-direction (see Figure 1). The other technique involves impacting the same point on the structure in the *z*-direction while moving a roving triaxial accelerometer (see Figure 2).

Question: **Do the measurements from these two testing techniques contain the same mode shape information?**

Answer: NO! The testing technique used in Figure 1 will yield mode shapes that only contain mode shape components in the *z*-direction except at point 3 which will have *x*, *y*, and *z* components. Whereas, the test setup in Figure 2 will yield mode shapes that have mode shape components in the *x*, *y*, and *z* directions for all 9 points of interest on the structure.

Now let's look at both of these test setups in detail. In Figure 1 we input a force in the *z*-direction while measuring the response due to that force in three directions at point 3. This allows us to measure three Frequency Response Functions (FRFs). Note that we would average several measurements while computing the FRFs. If a 4-channel analyzer is used, all three FRFs would be measured simultaneously. With a 2-channel analyzer the FRFs would be measured in a serial fashion. Thus, independent of how many channels your spectrum analyzer has, you could still make the measurements illustrated in both test setups. In Figure 1 as we rove the input force from one point to the next, we measure three FRFs. This would give us a total of 27 FRF measurements as the input is moved over the 9 points of interest.

The test setup in Figure 2 is similar to Figure 1 but definitely not the same. Here we always input the force at point 3 in the *z*-direction as we rove a triaxial accelerometer around to the points of interest. As mentioned above this set of FRF measurements could be made with either a 2 or 4 channel analyzer. For the test setup in Figure 1 we measure 3 FRFs at every point as we move the triaxial accelerometer around to the points of interest. This test also yields a total of 27 FRF measurements.

The differences between measurements taken from setups shown in Figures 1 and 2 can be explained from several points of view. Perhaps the best and least mathematical explanation is: *In making FRF measurements, only information about the motions at the two points in the measurement directions is obtained*. Remember that a frequency response function measurement is made between an excitation point and direction and a response point and direction. These are generally referred to as degrees-of-freedom (DOF) motions. Thus, in Figure 1 we see that we never made any measurements that were in the *x* and *y* directions except at point 3, where we made measurements in all three directions. The mode shapes analyzed from this set of measurements will show only motion in the *z* direction except for point 3 where (in general) it will have motion in all three directions. In Figure 2 we move a triaxial roving accelerometer and make measurements involving all three directions at each point. The mode shapes from this test will then (in general) have motion in all three directions at every point.

Another way of looking at the differences in these two testing situations is to use Maxwell's Reciprocity Relationship. To illustrate this, let's use the first three measurements we would make and then use Maxwell's relationship to better understand the motion we actually measured. From Figure 1 we would make the following three FRF measurements:

$$h_{3x,1z} \qquad h_{3y,1z} \qquad h_{3z,1z}$$

where $h_{3x,1z}$ was measured by impacting at 1*z* and measuring the response at 3*x*. Now if we apply Maxwell's Reciprocity Relationship (interchange excitation and response DOFs) involving the same three measurements we would then measure:

$$h_{1z,3x} \qquad h_{1z,3y} \qquad h_{1z,3z}$$

In other words, we now impact at point 3*x* and measure the response at 1*z* and then the next measurement would be with an impact at point 3*y* and again the response at 1*z*, etc. For each of these measurements we are always measuring the response in the 1*z* direction only and none of our measurements will contain information about 1*x* or 1*y*. This will continue to be the case for this test situation as we move around the structure which has motion only in the *z* direction.

Finally, we can show the differences in Figures 1 and 2 more mathematically. To do this we must first understand the relationship between what is called a *residue* and the *mode shape components* (amplitudes). The residue is a constant that is determined during the parameter estimation (curve-fitting) step of a modal analysis. The residue is part of a set of constants that is generally referred to as the *modal parameters*. The modal parameters for a particular mode of vibration consist of the mode's *frequency*, *damping* and *residue*. For the purpose of this discussion we can think of the residue as being related to the strength of a mode between two DOFs where the measurement was made for a particular mode of vibration.

It can be shown mathematically that a residue is proportional to the mode shape amplitude at the response DOF multiplied by the mode shape amplitude at the excitation DOF for a particular mode of vibration.

$$R_{ij}^{(k)} = u_i^{(k)} \cdot u_j^{(k)}$$

where:

$R_{ij}^{(k)}$ = the residue for mode *k* and DOF *i* and *j*

$u_i^{(k)}$ = the mode shape amplitude for mode *k* at DOF *i*

$u_j^{(k)}$ = the mode shape amplitude for mode *k* at DOF *j*

When we have completed a modal analysis and are viewing the results in animation, we are really looking at the set of all analyzed mode shape components (u) that make up the mode shape.

From Figure 1 we see we would estimate the following residues for a particular mode of vibration:

$$\begin{aligned}
R_{3x,1z} &= u_{3x} \cdot u_{1z} \\
R_{3y,1z} &= u_{3y} \cdot u_{1z} \\
R_{3z,1z} &= u_{3z} \cdot u_{1z} \\
&\vdots \\
R_{3x,3z} &= u_{3x} \cdot u_{3z} \\
R_{3y,3z} &= u_{3y} \cdot u_{3z} \\
R_{3z,3z} &= u_{3z} \cdot u_{3z} \\
&\vdots \\
R_{3x,9z} &= u_{3x} \cdot u_{9z} \\
R_{3y,9z} &= u_{3y} \cdot u_{9z} \\
R_{3z,9z} &= u_{3z} \cdot u_{9z}
\end{aligned}$$

To determine u from the residues we use the *driving point* residue $R_{3z,3z}$

$$u_{3z} = (R_{3z,3z})^{1/2}$$

Now that we have calculated u_{3z}, its value can be back substituted using the other residues to determine the other u values. However, when we inspect the u values that were calculated, we see that they are in the *z*-direction except for point 3 where they're in all three directions.

Now we take a look at Figure 2 and go through the same process as above. The residues for a particular mode in this case are:

$$R_{1x,3z} = u_{1x} \cdot u_{3z}$$
$$R_{1y,3z} = u_{1y} \cdot u_{3z}$$
$$R_{1z,3z} = u_{1z} \cdot u_{3z}$$
$$\vdots$$
$$R_{3x,3z} = u_{3x} \cdot u_{3z}$$
$$R_{3y,3z} = u_{3y} \cdot u_{3z}$$
$$R_{3z,3z} = u_{3z} \cdot u_{3z}$$
$$\vdots$$
$$R_{9x,3z} = u_{9x} \cdot u_{3z}$$
$$R_{9y,3z} = u_{9y} \cdot u_{3z}$$
$$R_{9z,3z} = u_{9z} \cdot u_{3z}$$

Again we take the square root of the driving point residue to, in this case, determine u_{3z}. Once we know u_{3z} it is substituted back to determine the rest of the u values. When we inspect the resulting mode shape components, we now see that we have mode shape amplitudes in all three directions at all of the points.

In summary, the main difference between the two testing techniques is that from Figure 1 only motion in the z-direction is measured (excluding point 3). Whereas, in Figure 2 our mode shapes would contain motion in all three directions. Also one might ask, "what did all those measurements in the first situation buy us." The same number of FRF measurements were made in both test setups and it seems that we got more information for the same number of measurements (27). If we go back and look at the u values that we calculated in the two setups, we see that in the first setup we have three different estimates of u in the z-direction at all the points, whereas in Figure 2 we only have one estimate in each of the three directions at all the points.

The testing technique illustrated in Figure 1 is the MRIT (Multiple Reference Impact Testing) technique. This method should be used whenever impact testing is performed with an analyzer that has more than 2 channels since the extra FRF measurements can be made without any penalties of time. These extra redundant measurements can be used by some modal analysis systems to better estimate the modal parameters. If you use the MRIT technique, it is not required that the extra reference transducers be at any particular points and/or directions. You should distribute the extra references around the structure in various directions. And, remember, that as you rove the input, it should in general be in all three directions.

Next month's Q&A column question: **When making an impact type of FRF measurement with several averages, is it important to hit the same point in the same direction for each average? If so, why?**

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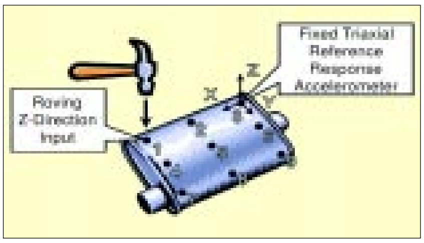

Figure 1. Fixed triaxial reference with roving Z-direction input.

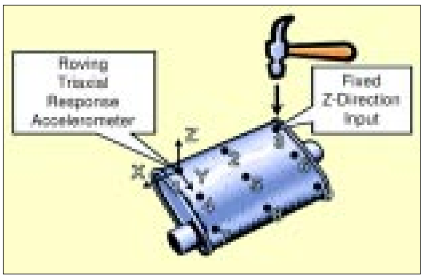

Figure 2. Roving triaxial response with fixed Z-direction input.